

**FURTHER ASPECTS OF THE COMPARISON
OF TWO GROUPS OF RANKED OBJECTS
BY MATCHING IN PAIRS¹**

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Suppose $\Gamma_X = (X'_{(1)}, \dots, X'_{(n)})$ and $\Gamma_Y = (Y'_{(1)}, \dots, Y'_{(n)})$ are two groups of stochastically ordered rv's, representing, say, the increasing strengths of the members of two chess teams. Let $\pi = (\pi_1, \dots, \pi_n)$ be a permutation of $(1, \dots, n)$. Then the statistic $S(\pi) = \sum_{i=1}^n I(Y'_{(i)} > X'_{(\pi_i)})$ measures the superiority of Γ_Y over Γ_X in matchings under π , where $I(y > x)$ is an indicator function. The dependence of $ES(\pi) = \sum_{i=1}^n P(Y'_{(i)} > X'_{(\pi_i)})$ on π , especially when $\pi = (1, \dots, n)$ and when π is randomly given, has been studied in Liu and David (1992) under two different models. After a review of the main results of that paper, some new optimality results are developed. In addition, a threshold model is used to treat tied comparisons.

1. Introduction

Tournaments in which n players or teams are compared by being matched up in pairs have been studied by mathematicians and statisticians at least since Zermelo (1929). The eminent author proposed and examined a method for evaluating the strengths of contestants in a round robin chess tournament that had to be broken off before each pair of players could meet. Independently, statisticians became interested through the connection between tournaments and the method of paired comparisons. In the latter, typically, n flavors are compared by being tasted in pairs, pairwise comparison providing maximal discrimination.

The method of paired comparisons goes back to the psychometrician Thurstone (1927), other notable early contributions being Kendall and Babington Smith (1940), Bradley and Terry (1952), and Kendall (1955). The last paper is perhaps the first to stress the connection between tournaments and paired comparisons, a point pursued in David (1959), where

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