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SOME REMARKS ON A NOTION OF POSITIVE DEPENDENCE, ASSOCIATION, AND UNBIASED TESTING[1]

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A new notion of positive dependence is studied. The new notion implies association of k random variables but is weaker than the notion of conditionally increasing in sequence.

1. Introduction

Tong (1990) discusses various notions of positive dependence of a collection of k random variables $(X_1, X_2, \ldots, X_k)$. Among the notions are multivariate totally positive of order 2 (MTP_2), conditionally increasing in sequence (CIS), and (positively) associated (A). In Proposition 5.1.2 on page 95 of Tong (1990), it is noted that

$$MTP_2 \Rightarrow CIS \Rightarrow A. \tag{1.1}$$

For applications it can be important to know whether a set of k random variables is MTP_2 or CIS, sometimes because the property implies A and sometimes because of other probability statements or inequalities that can be achieved. In Cohen, Kemperman, and Sackrowitz (CKS) (1992) another notion of positive dependence was introduced. This new notion, which we will call weak conditionally increasing in sequence ($WCIS$) is implied by CIS but implies A. Thus $WCIS$ is weaker than CIS but yet still $WCIS$ implies A. CKS (1992) used $WCIS$ to establish a class of unbiased tests for testing whether the natural parameters of k exponential family PF_2 distributions lie on a line against the alternative that the parameters are convex, i.e., their weighted second order differences are nonnegative.

Association is frequently a method of establishing unbiasedness of classes of tests. See for example, Perlman and Olkin (1980), Cohen and Sackrowitz (1987), and Cohen, Perlman and Sackrowitz (1991). CKS (1992) prove unbiasedness by establishing A via $WCIS$. In that study both MTP_2 and CIS fail to hold for the relevant variables.

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