Stochastic Inequalities
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STOCHASTIC INEQUALITIES FOR A REDUNDANCY ENHANCEMENT TO A SERIES OR PARALLEL SYSTEM

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We consider the question of where to allocate a redundant spare in a series or parallel system of components in order to stochastically optimize the resulting performance of the system. Both active (or warm or parallel) and standby (or cold) redundancy are considered. We show that if the components are stochastically ordered in the usual sense, then an active redundancy allocation to the weakest (strongest) component is stochastically optimal for a series (parallel) system. The situation is more delicate for standby redundancy. If the components are ordered according to the likelihood ratio ordering, then it is stochastically optimal to make a standby redundancy to the strongest component in a parallel system. It is shown however that even for this stronger sense of component ordering, the stochastically optimal redundancy allocation in a series system is not necessarily to the weakest component.

1. Introduction

We let $T_1, \ldots, T_n$ be random variables representing the lifetimes of n components which make up a series or parallel system. We will assume the lifetimes are independent, and that they are stochastically ordered (usually increasing) in some sense. There are two types of redundancy enhancements to the system that we consider: (1) an **active** (also called a **warm** or **parallel**) redundancy, and (2) a **standby** (also called a **cold**) redundancy. An active redundant spare is one which is actively working in parallel with one of the components in the system, while a standby spare is one which only begins to operate when the component for which it is 'standing by' ceases to operate. In any case the system performance as a whole is enhanced by a redundancy, and we will be interested in placing the redundant spare in the system so as to stochastically maximize its resulting lifetime. Consideration of an active redundancy leads one to study the maximum of random variables while that of a standby redundancy leads to the study of convolutions.

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