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SKEWNESS AND KURTOSIS ORDERINGS: AN INTRODUCTION

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Competing skewness orderings are surveyed. It is argued that those based on natural skewness functionals are preferable to those related to convex orderings. Analogous kurtosis orderings are also discussed. Here the role of convex and Lorenz orderings appears more natural.

1. Introduction

What is skewness? An analogous question regarding inequality led Dalton eventually down the majorization path via the enunciation of clearly agreed upon inequality reducing transformations. Can a similar analysis be performed with skewness? In a sense the answer is easy; skewness is asymmetry, plain and simple. It is of course easy to recognize symmetric distributions but not so easy to decide whether one non symmetric distribution is more unsymmetric than another. Robin Hood (i.e. rich to poor) transfers are at the heart of the accepted inequality orderings. It is natural to search for analogous basic operations which will increase skewness. The present paper surveys suggested skewness orderings (although not in the detail provided by MacGillivray (1986)) but puts its major focus on promoting a particular group of skewness orderings. Clear parallels may be discerned between some of these orderings and the Lorenz inequality ordering generated by Robin Hood operations.

What is Kurtosis? This is a bit harder. To quote our dictionary (Webster's of course) it is the state or quality of peakedness or flatness of the graphic representation of a statistical distribution. Again a plethora of competing orderings have been proposed (see Balanda and MacGillivray (1988) for a recent survey). Again we champion a particular ordering related to Lorenz ordering.

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