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INEQUALITIES FOR RARE EVENTS IN TIME–REVERSIBLE MARKOV CHAINS I

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The distribution of waiting time until a rare event is often approximated by the exponential distribution. In the context of first hitting times for stationary reversible chains, the error has a simple explicit bound involving only the mean waiting time ET and the relaxation time τ of the chain. We recall general upper and lower bounds on ET and then discuss improvements available in the case $ET \gg \tau$ where the exponential approximation holds. In a sequel, Stein's method will be used to get explicit bounds on the Poisson approximation for the number of non-adjacent visits to a rare subset.

1. Introduction

The Poisson approximation for numbers of rare events which actually occur, and the exponential approximation for the waiting time until first occurrence of a rare event, are useful throughout many areas on probability – one view of this big picture is presented in Aldous (1989). Here we study explicit bounds in these approximations, in the special setting of hitting times of stationary reversible Markov chains. This paper deals with the exponential approximation and bounds on the mean waiting time; a sequel (Aldous and Brown (1991)) studies Poisson approximations using an implementation of the Chen-Stein method.

The following set-up and notation will be used throughout. $(X_t; t \geq 0)$ is an irreducible finite-state reversible Markov chain in continuous time. The state space is I and the transition rate matrix is $Q = (q(i,j); i,j \in I)$ where $q(i,i) = -\sum_{j \neq i} q(i,j)$. Let π be the stationary distribution. The symmetrizable matrix $-Q$ has real eigenvalues $0 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \ldots$. Call $\tau = 1/\lambda_1$ the *relaxation time* of the chain. Let A be a fixed (proper, non-empty) subset of I, and let T_A be the first hitting time on A. So $0 < E_\pi T_A < \infty$.

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