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EXPECTATION INEQUALITIES FROM CONVEX GEOMETRY

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By making use of ideas from convex geometry, it is possible to derive novel inequalities for certain expectations. This is illustrated with reference to the Brunn-Minkowski inequality and the theory of zonoids.

1. Introduction. Among the various aspects of convex geometry, one which has a long and rich history is the study of inequalities. For a survey and an extensive bibliography, see Burago and Zalgaller (1988). Besides being of idepenent interest, many of these inequalities have been applied elsewhere. It is worth recalling that the second volume of Beckenbach and Bellman (1961) was to have been based on certain of these inequalities involving so-called mixed volumes. There have not, in fact, been many applications to probability and statistics, although there have been notable exceptions. The application of the Brunn-Minowski inequality to multivariate densities by Anderson (1955) is one example. Similarly, the fifty year old van der Waerden permanent conjecture was resolved by Egorychev (1981) by means which were originally developed by Alexandrov (see Burago and Zalgaller) (one should note the related, but *ad hoc* attack by Falikman, 1981). These tools have also been used by Stanley (1981) to resolve certain combinatorial questions. As indicated by these successes, it seems worthwhile to look for other connections between convex geometry and problems of a stochastic nature. The purpose here is to survey some possibilities, the very last section of the paper devoted explicitly to a novel stochastic ordering. We shall keep the discussion informal and largely omit proofs, which appear elsewhere.

In the next section we present notation and preliminaries. Section 3 is devoted to a general form of the Brunn-Minkowski inequality and its relation to Anderson's inequality. Section 4 discusses inequalities for random determinants. The last section treats an open question on the nature of a certain class of convex bodies.

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