Stochastic Orders and Decision under Risk
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STOCHASTIC ORDERS AND COMPARISON OF EXPERIMENTS

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Exploring criteria for majorization, exact and approximate, univariate and multivariate, we relate them to criteria for information orderings of statistical experiments. After providing some basic criteria for comparison of experiments, we observe their straightforward generalizations to general families of measures. Thus LeCam's randomization criterion extends to a criterion for comparing families of measures. Reversing the randomizations, we obtain dilation like kernels mapping densities, exactly or approximately, into densities.

Using this, we derive criteria for comparison of measures in terms of integrals of given functions. In particular we obtain well-known criteria for one measure being a dilation of another measure and for stochastic orderings of distributions on partially ordered sets.

Experiments having two point parameters sets, i.e. dichotomies, enjoy a variety of striking properties which are not shared by experiments in general. Dichotomies may be studied in terms of their Neyman-Pearson functions, which are functions describing the relationships between the probabilities of errors of the two kinds for most powerful tests. These functions are the inverses of the Lorenz functions of econometrics. Observing this, we readily obtain various criteria for one distribution being approximately Lorenz majorized by another.

1. Introduction. Majorization and Comparison Experiments.

The purpose of this paper is to discuss relationships between developments within the theory of comparison of statistical experiments on the one hand, and various notions of "stochastic" orders on the other. As we shall see, the theory of comparison of experiments not only throws light on standard notions of stochastic order, but also provides interesting generalizations of well-known results.

The paper provides the required results from the theory of statistical experiments. However proofs are often incomplete. The reader who wants

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