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REGULAR, SAMPLE PATH AND STRONG STOCHASTIC CONVEXITY: A REVIEW

BY MOSHE SHAKED* AND J. GEORGE SHANTHIKUMAR

University of Arizona and University of California, Berkeley

Several notions of stochastic convexity and concavity and their properties are described in this survey. The notion of sample path stochastic convexity is a refinement of the well used notion of stochastic ordering, and it can be used to construct, on a common probability space, random variables which have desirable convexity (or concavity) properties with probability one.

Three open problems from the literature are described. These problems could not be resolved until the introduction of the stochastic convexity notions which are described in this survey. The solutions of these problems illustrate the strength and the usefulness of these notions. Each notion is accompanied by a description of some of its applications. References for more detailed study of these notions are given. Indications of further work in this area are included.

1. Introduction. Regular, Sample Path and Strong Stochastic Convexity notions are very valuable in many areas in probability and statistics such as queueing and reliability theory. Consider, for example, the following three open problems:

PROBLEM 1. Consider a single stage queueing system at which customers arrive according to a doubly stochastic Poisson (DSP) process. The stochastic intensity of the DSP is a Markov process on $\{\lambda_1, \lambda_2\}$ $(\lambda_i \geq 0, i = 1, 2)$. The expected time this Markov process spends in state λ_i is $\theta r_i, i = 1, 2$ for some $r_i \geq 0, i = 1, 2$, and $\theta \geq 0$. The service times of the customers are independent and identically distributed random variables. Let $EW(\theta)$ be the average work load in this DSP/G/1 queueing system.

CONJECTURE 1. (Ross 1978). $EW(\theta)$ is a decreasing function of θ.

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