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BOUNDS FOR DISTRIBUTIONS WITH MULTIVARIATE MARGINALS

BY LUDGER RÜSCHENDORF

University of Münster

Some explicit results and bounds are derived for integrals of functions on product spaces assuming the knowledge of certain multivariate marginals of the underlying distribution. One class of bounds is obtained by using a general reduction principle and the relation to Bonferroni type bounds. A further method is to reduce the problem by conditioning to a problem with simpler marginal constraints. It is proved that one can reduce general decomposable marginal systems to the case of series and star-like systems. For this reason, series and star-like systems are given special consideration. For some nonregular systems, one can derive good bounds by considering all regular subsystems. This method implies, in particular, a characterization or the marginal problem for a circle system of marginals. The special queston of construction of optimal couplings for random vectors w.r.t. the L_p-distance is discussed. This question is related to the investigation of sharp inequalities of the type $f(x) + g(y) \leq |x - y|^p$. Finally, a combinatorial application is given to the support of multidimensional permutation matrices.

1. Introduction. The formal definition of the model with multivariate marginals is the following. Let $S = S_1 \times \cdots \times S_n$ be the product of n Borel spaces with σ-algebra $\mathcal{B} = \bigotimes_{i=1}^n \mathcal{B}_i$, $\mathcal{B}_i$ the Borel σ-algebras on S_i. Let $\mathcal{E} \subset \mathcal{P}\{1, \cdots, n\}$, the system of all subsets of $\{1, \cdots, n\}$, with $\bigcup_{J \in \mathcal{E}} J = \{1, \cdots, n\}$ and let for $J \in \mathcal{E}$, $P_J \in M^1\left(\prod_{j \in J} S_j\right)$ be a consistent system of multivariate distributions on $\pi_J(S) = \prod_{j \in J} S_j =: S_J$, π_J being the J-projection from S to S_J and $M^1(S_J)$ denoting the set of all probability measures on S_J. Consistency means that $J_1, J_2 \in \mathcal{E}$, $J_1 \cap J_2 \neq \emptyset$ implies that $\pi_{J_1 \cap J_2} P_{J_1} = \pi_{J_1 \cap J_2} P_{J_2}$. Define

$$M_{\mathcal{E}} := M(P_J, J \in \mathcal{E}) \tag{1}$$

to be the set of all probability measures on S with marginals P_J, $J \in \mathcal{E}$.

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