Stochastic Orders and Decision under Risk
IMS Lecture Notes – Monograph Series (1991)

SOME THEORY OF STOCHASTIC DOMINANCE

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Three different types of stochastic dominance relations are considered: set dominance, kernel dominance and higher degree dominance. The connections between these definitions are examined. Preservation results are given and implications between joint and marginal dominance are studied in the finite and infinite dimensional setting.

1. Introduction. For distributions on the real line, two basic stochastic orderings have been of interest to researchers in many fields: stochastic dominance with respect to all increasing functions and stochastic dominance with respect to all convex functions. These orderings can be characterized by shifts and dilations, respectively, or, in the first case, by inequalities for distribution functions, and, in the second case, by inequalities for integrals of the distribution functions. Beginning with these characterizations several attempts have been made to unify the theory of stochastic dominance relations in d-dimensional and more general spaces (Brumelle and Vickson (1975), Fishburn and Vickson (1978), Stoyan (1977) (1983), Mosler (1982)). Based on this tradition, the primary aim of this paper is to investigate three different ways by which stochastic dominance relations on several spaces may be characterized. The first one is the characterization via probability inequalities for certain families of sets. Orderings which allow for this kind of characterization are named set dominance orderings. The second approach employs Markov kernels to define a stochastic ordering. These orderings are called kernel dominance orderings. Third, inequalities on integrals of distribution functions are

* The authors thank Yosi Rinott for processing this paper and Bruno Bassan and two anonymous referees for their valuable comments.

† This paper was written while the second author was a member of GNAFA-CNR.

AMS 1980 Subject Classification: 90A05, 90A10, 60E15

Key words and phrases: Set dominance, kernel dominance, higher degree stochastic dominance, preservation of stochastic orderings, copula, dominance for stochastic processes