STOCHASTIC ORDERING FOR MARKOV PROCESSES ON PARTIALLY ORDERED SPACES WITH APPLICATIONS TO QUEUEING NETWORKS

By William A. Massey

AT&T Bell Laboratories

The state spaces for queueing networks are intrinsically multidimensional. As a result, a theory of stochastic ordering for Markov processes on partially ordered spaces is a natural setting for the formulation of comparison theorems of queueing networks. The partially ordered structure of the state space gives rise to a variety of stochastic orders, that are distinct only when the space is *not* totally ordered. In particular, we can define stochastic orderings that are not equivalent to sample path comparisons. For all these orderings, we give a unified theory that allows us to compare Markov processes on such spaces, given their initial distributions and infinitesimal generators. Examples will be given to show how these results apply to queueing networks. In addition, we will give a simplified proof for a special case of Strassen's theorem as it applies to a sample path comparison of random variables.

0. Introduction. Queueing network theory has applications to such diverse areas as data communications, manufacturing, highway traffic, and population biology. Abstractly, we can view a queueing network as a collection of *sites* and *particles*. Sites can be thought of as nodes in a network, and particles as customers for the system. Each object has its own set of instructions as to how to move from one site to another, and how long to stay at any given site. The theory that develops is the method by which we analyze the flows between sites. In particular, we may have interest in how long any given object may take to traverse these sites, which is better known as the sojourn time. We may also want to know how many particles may accumulate at various sites, which we call the *queue length vector process*. It is the latter on which we will focus our attention.

Since we never have precise information on the arrival, service, or transfer patterns for particles in these networks, we are naturally led to the use of