Stochastic Orders and Decision under Risk
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ORDERING REGRESSION MODELS OF GAUSSIAN PROCESSES

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We study information orderings of regression models of continuous time Gaussian processes. This is done in the framework of "comparison of experiments." The theory includes parametric, nonparametric and semiparametric models. Particular simple criteria for information orderings are obtained for linear models. Applications include the design problem for regression models with correlated Gaussian errors and the information contained in additional observation periods. Several examples are discussed.

1. Introduction. Informational inequalities between models of continuous time Gaussian processes are investigated. Let

$$X(t) = S(\theta, t) + Z(t), \quad t \in I, \tag{1.1}$$

where I is a compact subset of $\mathbb{R}$, Z is a zero-mean Gaussian process with covariance function K, θ is an unknown parameter belonging to an arbitrary parameter set Θ, and $S : \Theta \times I \to \mathbb{R}$ is a deterministic function. K and S are assumed to be known. We are interested in comparing the information contained in such models concerning inference about θ when Θ is fixed while S, K and I may vary from model to model. The basic ordering which will be considered is the same as the stochastic ordering of likelihood processes defined by convex functions.

The above model is an example of a signal-plus-noise model with deterministic signal S and noise process Z. Linear models of type (1.1) are given, for instance, by parametric regression models

$$X(t) = \sum_{i=1}^{r} \theta_i h_i(t) + Z(t),$$

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