Stochastic Orders and Decision under Risk
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ORDERINGS OF RISKS AND THEIR ACTUARIAL APPLICATIONS

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In actuarial theory a risk is a random variable describing a claim size (a single claim size, or the total claim amount of one contract in one period, or the aggregate claim of a portfolio of contracts in one period, e.g.). In the present contribution a number of (well-known as well as new) orderings of random variables are discussed. In particular, the relations between these orderings are investigated, and interpretations in terms of actuarial applications are given. Furthermore, the stability of the orderings with respect to convolutions and the forming of random sums is examined. Finally, it is shown that this approach can be used to generate formulas for risk premiums.

1. Introduction. The starting point of the present paper is given by the following two questions which are closely associated. First, is there a specific risk for which insurance companies are exposed to but for which companies in other economic branches are not (a "technical" or "actuarial" risk)? And if so, how can it be quantified? Second, how can the "dangerousness" of a risk (i.e., a claim variable) be described in the models of risk theory? While a treatment of the first problem involves economic aspects and goes beyond the scope of the present paper, the second question leads to the introduction of quantities and functions which induce order relations in the set of random variables describing risks.

An appropriate framework for such a study is given by the probabilistic models and methods of risk theory. Therefore, we start with a brief review of the so-called collective theory of risks. Subsequently, (first order) stochastic dominance and four order relations associated with stochastic dominance are introduced. For each of these orderings, interpretations in terms of actuarial applications are given. For specific distributions which are frequently applied in actuarial mathematics, necessary and sufficient conditions for domination in terms of the distribution parameters are derived.

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