

REPRESENTATION THEOREMS FOR
MEASURES OF LOCATION
AND FOR MEASURES OF DISPERSION

BY ALESSANDRA GIOVAGNOLI AND GIULIANA REGOLI

University of Perugia

In this paper we characterize classes of statistical functionals through some results which have been inspired by a classical theorem on mean values due to de Finetti, Nagumo, and Kolmogorov. All the functionals are order preserving w.r.t. particular stochastic orderings.

The first result is a quasi-linear representation for functionals assuming, together with monotonicity w.r.t. first degree stochastic dominance and associativity, a particular continuity condition which can be interpreted as a mild type of robustness. This result is used in a “dual” way to characterize other measures of location, like median, quantiles, trimmed means, and Winsorized means.

In the second part of the paper our aim is to characterize some measures of dispersion of a distribution around its expected value which are order preserving w.r.t. the so-called dilation ordering. Most statistical indices of variability can be obtained in this way. This and a “dual” theorem also account for several measures of inequality, which are order preserving with respect to the concentration ordering based on the Lorenz curve, like Gini’s celebrated index.

1. Introduction. In 1930 Nagumo and Kolmogorov (independently) and later de Finetti proved a classical theorem which characterizes suitable means of n real numbers (in Nagumo’s and Kolmogorov’s versions) and of distributions with support in an interval (in de Finetti’s) as quasi-linear functions, in fact 1-1 transformations of expected values. In a decision theory context the dFNK theorem states necessary and sufficient conditions in order to represent the certainty equivalent of a monetary lottery as a transformed expected utility, where $u(\cdot)$, the utility function, is increasing. This theorem has been extended in many ways (see for instance, Chew, 1983 and 1989, and Fishburn, 1986).

A crucial assumption of dFNK is associativity which in the parallel development of expected utility theory is replaced by a stronger requirement, the

AMS 1980 Subject Classification: Primary 62C05; Secondary 90A43, 62F35.

Key words and phrases: Dilation ordering, L -functionals, Lorenz ordering, quasi-linear means, stochastic dominance.