Stochastic Orders and Decision under Risk
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STOCHASTIC ORDERINGS IN RELIABILITY

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In this paper we give a short survey of topics in reliability and show how stochastic ordering can be used in reliability theory: to describe aging, to compare performance processes, and to find optimal replacement policies.

1. Introduction. Examples of the use of stochastic orderings in reliability are numerous. Therefore, we do not attempt to give a survey of the field, but we only describe some main ideas on how to use stochastic orderings in reliability theory. It should not be unreasonable to expect that readers of this paper know the fundamentals of stochastic ordering as can be found e.g. in Mosler (1982) and Stoyan (1983), but perhaps they are not already familiar with the subject of reliability theory. Therefore we describe in Section 1.1 a well known system with maintenance through spares and repair in order to illustrate the typical topics and problems of reliability theory. After the discussion of some problems which show that stochastic ordering is a useful tool in reliability theory, we give in Section 1.2 a detailed description of the further contents of the paper; aging, comparison of processes, optimal maintenance through replacement. Section 1.3 gives the notation we will use for the different kinds of stochastic ordering in Section 2. After introducing the main concepts of aging, we give a summary of their generalization to the multicomponent case and to the use of more complicated information about the system. In Section 3 we give a detailed description of the performance process of our standard reliability system and state sufficient conditions that allow the stochastic comparison of standard reliability processes. We compare these conditions with the conditions given by Shaked and Shantikumar (1988) for the more special case of dependent coupled alternating renewal processes. In Section 4 we give an example to show how stochastic ordering is useful, if

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