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MULTIVARIATE PROBABILITY INEQUALITIES: CONVOLUTION THEOREMS, COMPOSITION THEOREMS, AND CONCENTRATION INEQUALITIES

BY MORRIS L. EATON* AND MICHAEL D. PERLMAN†

University of Minnesota and University of Washington

Several important multivariate probability inequalities can be formulated in terms of multivariate convolutions of the form $\int f_1(x)f_2(x-\theta)dx$, where usually $f_1 = I_C$ is the indicator of a region $C \subseteq \mathbb{R}^n$, f_2 is a probability density on $\mathbb{R}^n$, and θ is a translation parameter. Often f_1 and f_2 possess convexity, monotonicity, and/or symmetry properties. More general multivariate compositions of the form $\int h(x)f(x \mid \theta)\mu(dx)$ also arise. Here several important convolution and composition theorems will be reviewed; these provide comparisons of Prob(C) under differing multivariate distributions. The convolution theorems are then applied to obtain concentration inequalities for Prob(C) under Gaussian or elliptically contoured distributions with varying multivariate scale parameter Σ.

1. Introduction. In multivariate statistical analysis, the power function of a hypothesis test and the confidence coefficient of a confidence region are determined by the probability of a multivariate region $C \subseteq \mathbb{R}^n$. Frequently the region C possesses convexity, monotonicity, and/or symmetry properties inherited from corresponding properties of the multivariate distributions in the statistical model. In order to establish properties of the statistical procedure such as unbiasedness, it is necessary to compare the probabilities of C under different multivariate distributions in the model.

Several important multivariate probability inequalities can be formulated in terms of *convolutions* of the form

$$\psi(\theta) \equiv \int_{\mathbb{R}^n} f_1(x)f_2(x-\theta)dx. \tag{1.1}$$

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