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SECOND ORDER BONFERRONI-TYPE, PRODUCT-TYPE AND SETWISE PROBABILITY INEQUALITIES

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This review paper considers a variety of lower bounds to $\text{Prob}(X_1 \leq c, \cdots, X_n \leq c)$ involving one and two-dimensional marginal probabilities. Some of these bounds, e.g., Bonferroni-type, do not require conditions on the random variables. Others of these inequalities, e.g., product-type, require positive dependence conditions on $X_1, \cdots, X_n$ for the inequalities to hold. Because all of the two-dimensional bounds depend on the labeling order of the random variables, various permutation-optimized versions of the bounds are described. Relationships among the various bounds are also considered.

1. Introduction and Overview. In many statistical applications such as moving window detection and the calculation of the expected stopping time and its variance in sequential analysis, one wishes to find a constant c such that

$$P(c) = \text{Prob}(X_1 \leq c, \cdots, X_n \leq c) = 1 - \alpha, \tag{1.1}$$

where $\boldsymbol{X} = (X_1, \cdots, X_n)$ is a vector of dependent random variables and $0 \leq \alpha < 1$ is given. The constant c which satisfies (1.1) is called the *multivariate α-level critical value.* Exact calculation of c can entail extensive computation. Sometimes the calculation of c is intractable (even with the use of a computer) because it requires iteration and high dimensional multivariate numerical integration. This is particularly true in the types of applications noted above. Due to this dilemma, which is known as the "curse of dimensionality," there has been considerable interest in finding easily computable approximations to $P(c)$ which can be used in an iterative search procedure to find an approximation to the multivariate α-level critical value.

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