Stochastic Orders and Decision under Risk
IMS Lecture Notes – Monograph Series (1991)

STOCHASTIC ORDERS AND THEIR APPLICATION TO A UNIFIED APPROACH TO VARIOUS CONCEPTS OF DEPENDENCE AND ASSOCIATION

BY REINHARD BERGMANN

Rheinfelden

Multivariate stochastic partial orderings are studied, especially in the context of probability inequalities. Extensions of stochastic orderings to the multicomponent case for general product spaces are developed. They provide a sound basis for a unified representation of dependence and association notions.

1. Introduction. Stochastic orderings have found a wide field of application in probability, statistics, and statistical decision theory, see Stoyan (1983), Mosler and Scarsini (1991), as comprehensive references. In probability theory, they are useful in deducing probability inequalities, comparing stochastic models, establishing bounds and inequalities in reliability and queueing theory, in statistics for example in hypothesis testing, simultaneous comparisons, multiple decision problems, and in economics in decisions under risk, particularly in multi-attribute utility theory.

The approach in this paper is mainly to define various stochastic orderings starting from interesting multivariate probability inequalities. To characterize the stochastic orderings, several quite different equivalent conditions are given. The stochastic orderings are associated with inequalities between expectations of functions with respect to the corresponding distributions or random variables. A very interesting and important problem is to find the class of functions which implies the inequality between the expectations. For this issue solutions are given.

We prefer the presentation in terms of random variables rather than of distribution functions or probability measures in order to facilitate intuitive handling of the inequalities. Consider a partially ordered measurable space $(E, \leq)$ and random variables X, Y with values in E.

AMS 1980 Subject Classification: Primary 60E15; Secondary 62H20

Key words and phrases: Multivariate stochastic orders, probability inequalities, dependence, association.