Stochastic Orders and Decision under Risk
IMS Lecture Notes – Monograph Series (1991)

LORENZ ORDERING OF ORDER STATISTICS

BY BARRY C. ARNOLD AND JOSE A. VILLASEÑOR

University of California, Riverside, and Colegio de Postgraduados, Chapingo

The Lorenz order is a natural tool for comparison of the variability of non-negative random variables. In many contexts (especially reliability) variability comparison of order statistics is of interest. A survey is provided of available results regarding the Lorenz order of order statistics.

1. Introduction. Let $X_{i:n}$ denote the ith order statistic from a sample of size n from the distribution $F_X (i = 1, 2, \ldots, n;\ n = 1, 2, \ldots)$. Yang (1982) and David and Groeneveld (1982) have identified certain situations in which the $X_{i:n}$'s are ordered with respect to variability as measured by the variance. In many contexts, a more basic variability ordering is that provided by the Lorenz order. In a reliability context, the lifetime of a k out of n system is $X_{n-k+1:n}$, the waiting time until less than k components are still functioning. Attention is focussed on controlling not only the mean life but also the variability. Predictable life length is desirable. The engineering may choose how to build his k of n system out of exchangeable components. He is consequently concerned with choosing k and n to reduce variability in the life length, subject to given mean life constraints; a natural scenario for Lorenz order comparisons.

Lorenz ordering relations among uniform order statistics are first developed. Analogous relations are developed involving more general common distributions for the X's. We begin with a brief survey of characterizations of the Lorenz order.

2. The Lorenz Order. A convenient survey of relevant material is available in Arnold (1987).

Let $\mathcal{L}$ denote the class of all non-negative random variables with finite positive expectations. For a random variable X in $\mathcal{L}$ with distribution function

---

AMS 1980 Subject Classification: primary 60E05; secondary 62G30

Key words and phrases: Variability comparisons, density crossings, uniform order statistics, Lorenz curve.