Stochastic Orders and Decision under Risk
IMS Lecture Notes – Monograph Series (1991)

PRESERVATION AND ATTENUATION OF INEQUALITY AS MEASURED BY THE LORENZ ORDER

By Barry C. Arnold

University of California at Riverside

The Lorenz order is defined on the class of all non-negative random variables with positive finite expectations. Interest has often focussed on characterization of transformations, defined on the space of such random variables, which either preserve or attenuate inequality. Results of this genre involving deterministic and random transformations are surveyed. In some settings distributions are changed by weightings (in the sense of Rao (1965)) or by mixing, rather than by transformations. Preservation and attenuation results in such scenarios are summarized.

1. Introduction. Let $\mathcal{L}$ denote the class of all non-negative random variables whose expectations exist and are strictly positive. With any random variable X in $\mathcal{L}$ with distribution function F_X, there is associated a Lorenz curve L_X defined by

$$L_X(u) = \int_0^u F_X^{-1}(s)ds \Big/ \int_0^1 F_X^{-1}(s)ds, \quad u \in [0,1] \tag{1.1}$$

where $F_X^{-1}(s) = \sup\{x : F_X(x) \leq s\}$. This definition of the Lorenz curve can be traced back explicitly to Gastwirth (1971). The Lorenz partial order on $\mathcal{L}$ denoted by $\leq_L$ is defined by

$$X \leq_L Y \Longleftrightarrow L_X(u) \geq L_Y(u) \;\forall\; u \in [0,1]. \tag{1.2}$$

If $X \leq_L Y$ then we say that X exhibits no more inequality than does Y. We define the strong Lorenz order, $X <_L Y$ by

$$X <_L Y \Longleftrightarrow X \leq_L Y \text{ and } Y \not\leq_L X,$$

AMS 1980 Subject Classification: Primary 62P20; Secondary 60E05

Key words and phrases: Weighted distributions, mixtures, Lorenz curve, random taxation, misreported income, majorization.