

STOCHASTIC ORDER AND MARTINGALE DYNAMICS IN MULTIVARIATE LIFE LENGTH MODELS: A REVIEW

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The purpose of this paper is to review some ideas connected to aging and dependence, in the context of technical reliability. The dynamic aspects of these notions are stressed throughout. The review is based mainly on the authors' own work during the past decade, but it connects very closely with some recent results by Shaked and Shanthikumar. New definitions, results and examples are also presented.

1. Introduction and Mathematical Preliminaries. In this paper we review some notions of aging and dependence which arise naturally in the context of engineering reliability. These notions are based on “ordinary” stochastic order of multivariate distributions in the positive orthant. But the given definitions differ in two important respects from the standard comparison of multivariate distributions with respect to stochastic order: they are *dynamic* and *conditional*.

The first characterization means that *time* becomes a key element of our analysis. Time is of course present in every meaningful notion of aging. But it is equally basic in every causality reasoning, and therefore also enters our modeling of dependence.

The second characterization emphasizes the role of *information*, which corresponds to the observed behavior of the considered device in the past and forms a natural basis on which its future behavior can be predicted.

From a mathematical point of view our approach to modeling aging and dependence can be seen as a particular application of the modern stochastic calculus and martingale theory for point processes. As a consequence, our presentation is somewhat unusual in the reliability tradition. However, we

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