Stochastic Orders and Decision under Risk
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REMARKS ON A RANDOM SURFACE

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A simple discrete random surface is defined. Its stochastic ordering/ inequality properties are discussed and some open problems are presented.

In this paper we discuss a simple discrete random surface introduced within a statistical mechanics context in [AN1, AN2]. Our purpose here is to survey some stochastic ordering/inequality properties and some easily stated open problems. For the sake of simplicity, we will mainly deal with a limiting case (corresponding to infinite temperature) of the model treated in [AN1, AN2]. We begin by discussing some of the physical motivation behind such random surface models. For more physical background and for other random surface models, see the papers in [DD] and the references in [AN1, AN2].

Consider a flat horizontal smooth solid substrate, in thermal equilibrium at temperature T, with two immiscible fluids lying above it – one a liquid labelled A (e.g., a lubricant) and the other a gas labelled B (e.g., air). It can happen that above some temperature T_w, there is a macroscopic slab of A between the substrate and B, while below T_w, A is squeezed out (or is of microscopic thickness). Above T_w, one says that A wets the substrate perfectly, and the transition at T_w, is known as a wetting phase transition. To model this phenomenon, one may regard the interface between fluids A and B as a two-dimensional surface and postulate an energy function E on some space of allowed configurations of the surface. At temperature T, the surface is random with a probability density proportional to $\exp(-E/T)$. When $T = \infty$, all allowed configurations are equally likely (see property (v) below).

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