U-STATISTICS AND DOUBLE STABLE INTEGRALS

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Summary

We derive the tail behaviour of the double stable integral

$$I(h) = \int_0^1\int_0^1 h(x,y)X(dx)X(dy),$$

where X is a completely asymmetric stable process.

1. Introduction. First we shall show the relation between the double stable integral and a simple U-statistic. Let $\{X(t): 0 \leq t < \infty\}$ be a completely asymmetric stable process with characteristic exponent $\alpha \in (0,1)$ and $\beta = 1$. For the theory of stable distributions we refer to Gnedenko-Kolmogorov [GK 54], Breiman [Bre 68] or Feller[Fel 71]. A summary can be found in Mijnheer [Mij 75]. We use the notation as used in [Mij 75]. See [Mij 75] section 3.2 for a review of properties of stable processes. The random variables X_i, $i = 1, 2, \ldots$ are i.i.d. and have the same distribution as $X(1)$. $X \stackrel{d}{=} Y$ means that X and Y have the same distribution. $X \in D(\alpha, \beta)$ (resp. $D_N(\alpha,\beta)$) means that X belongs to the domain of (resp. normal) attraction of the stable distribution with parameters α and β. Then we have

$$n^{-2/\alpha}\sum_{\substack{i=1\\ i\neq j}}^{n}\sum_{j=1}^{n} X_iX_j \stackrel{d}{=} n^{-2/\alpha}\sum_{i\neq j}\sum \{X(i)-X(i-1)\}\{X(j)-X(j-1)\}$$

$$\stackrel{d}{=} \sum_{i\neq j}\sum \{X(in^{-1})-X((i-1)n^{-1})\}\{X(jn^{-1})-X((j-1)n^{-1})\}.$$

This quadratic form is in a natural way related to the double stable integral

$$I(h) = \int_0^1\int_0^1 h(x,y)X(dx)X(dy) \tag{1.1}$$

where the function h is given by

$$h(x,y) = \begin{cases} 1 & 0 \leq x,y \leq 1 \text{ and } x \neq y \\ 0 & \text{otherwise}. \end{cases} \tag{1.2}$$