MEASURE-VALUED PROCESSES: TECHNIQUES AND APPLICATIONS

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Abstract

Several techniques are presented for the analysis of measure-valued stochastic processes. These methods are then applied to a number of examples so as to determine the behavior of the processes at fixed times, in the long term, and in the renormalization limit.

I. Introduction. Interest in the theory of measure-valued stochastic processes, which were first introduced by Dawson (1975) in the study of branching diffusion systems, has recently increased as a result of the work of Dynkin (1988, 1989) and Perkins (1988, 1989, 1990) on superprocesses. In the interim, measure-valued processes have been used to describe the dynamics of populations whose underlying distributions are continuously changing, and which are therefore described via a distribution or random measure at each fixed time. They also arise as the diffusion approximation to certain real-valued processes describing spatially-distributed systems. Applications that lead to measure-valued processes in the diffusion limit include models that describe the behavior of systems of branching and diffusing particles (Dawson (1977), Dawson-Hochberg (1979), Hochberg (1980, 1983), Iscoe (1986, 1988), Dawson-Iscoe-Perkins (1989)); models describing frequency distributions of alleles in neutral, non-neutral and interactive populations (Fleming-Viot (1979), Dawson-Hochberg (1982, 1983), Hochberg (1986), Ethier-Kurtz (1987), Ethier-Griffiths (1987, 1990), Vaillancourt (1990a,b)); and the continuous limit of hierarchically-structured branching and branching diffusion systems (Dawson-Hochberg-Wu (1990), Dawson-Hochberg (1991)).

In what follows, we review several techniques for analyzing measure-valued stochastic processes, present examples in which such processes arise, and show how these techniques are used to study the behavior of such processes at fixed times, in the long term, and in the renormalization limit.

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