EXTINCTION PROBABILITIES OF BRANCHING PROCESSES IN RANDOM ENVIRONMENTS

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Abstract

In the supercritical branching process with independent and identically distributed environments, it is shown that under certain regularity conditions there exists a parameter $\theta_0 > 0$ such that the probability of extinction starting with k individuals, q_k, is asymptotically of order not less than $k^{-\theta_0}$ and of smaller order than $k^{-\theta}$ for any $\theta < \theta_0$. An application to the optimal choice of strategy for minimizing the probability of extinction is mentioned.

1. Introduction and Statement of Results. We consider a branching process $\{Z_n; n = 0,1,2...\}$, where Z_n denotes the population size at time n. Reproduction is affected by a sequence of environment variables $\bar{\zeta} = \{\zeta_0,\zeta_1,\zeta_2,...\}$ in the following way: for each n, conditional on $\bar{\zeta}$ and $Z_0, Z_1, ... Z_n$, the family sizes of the Z_n individuals at time n are independent random variables each with a distribution which is determined by ζ_n, and whose probability generating function (p.g.f.) we shall denote by ϕ_{ζ_n}. Then Z_{n+1} is just the sum of these family sizes.

Particular models for the environment variables are a sequence of independent identically distributed (i.i.d.) random variables (Smith and Wilkinson [4]) and, more generally, a stationary ergodic sequence (Athreya and Karlin [1]). In this paper, we consider only the Smith-Wilkinson model, although subsequent work has generalized the results to certain types of Athreya-Karlin model.

Let $q(\bar{\zeta})$ be the probability, conditional on $\bar{\zeta}$, that the population becomes extinct starting with a single ancestor:

$$q(\bar{\zeta}) = P(Z_n \to 0 \text{ as } n \to \infty | \bar{\zeta}, Z_0 = 1).$$

Then, because, conditional on the environment sequence, lines of descent are independent, the unconditional probability of extinction starting with k ancestors is

$$q_k = E[q(\bar{\zeta})^k]$$