The Genealogy of Patterns of ESS's

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Summary

In a finite conflict a given payoff matrix **A** may possess **ESS** 's with supports $T_1, T_2, ..., T_k$. Such a set of supports is called a pattern and is attainable since there exists a payoff matrix **A** with **ESS** 's with those supports. This paper presents a summary of previous work aimed at specifying the set of attainable patterns, the most recent results for conflicts with 5 pure strategies and defines the notion of a genealogy of patterns. This genealogy, in which a subpattern is regarded as an 'offspring' is displayed for n = 4 and n = 5 where the genealogy has been produced so as to minimize the number of line-intersections using simulated annealing.

Introduction. A finite conflict is defined by a pair $\{\mathbf{U}, \mathbf{A}\}$ where $\mathbf{U} = \{1, 2, 3 ...n\}$ is the strategy space i. e. there are n pure strategies labeled 1 through n, and **A** is an $n \times n$ matrix whose elements a_{ij} are the payoffs; a_{ij} is the payoff to an individual who plays strategy i and whose opponent plays strategy j. We consider, as in classical game theory (Von Neumann and Morgenstern,1953), the mixed extension so that the strategies are $\mathbf{p} = (p_1, p_2 ... p_n)$ where $p \in \Delta, \Delta = \{p; p \geq 0, \Sigma p_i = 1\}$. The payoff for strategy **p** against **q** has the expected value $\mathbf{p}^T \mathbf{Aq}$ and this is also the payoff of strategy **p** in a population which is playing **q** on average.

An **ESS**, evolutionarily stable strategy, is a **p** such that for every $\mathbf{q} \neq \mathbf{p}$

(1)$\mathbf{E(p, p)} \geq \mathbf{E(q, p)}$ and

(2) if equality in (1) then $\mathbf{E(p, q)} > \mathbf{E(q, q)}$.

For the generic case of a finite conflict, a strategy **p** is an **ESS** iff (if and only if) $\mathbf{E}(i, \mathbf{p}) = \mathbf{E(p, p)}$ all $i \in \mathbf{R(p)}$, where $\mathbf{R(p)} = \{i: p_i > 0\}$ the support of **p**, $\mathbf{E}(j, \mathbf{p}) < \mathbf{E(p, p)}$, all $j \notin \mathbf{R(p)}$, and if $\mathbf{B} = (a_{ij})$, $i, j \in \mathbf{R(p)}$ then $\mathbf{z^tBz}$ is negative definite where z such that $\Sigma z_i = 0$, Haigh(1975).

If **P** is the power set of **U** and $\mathbf{T} = \{T_1, T_2 ... T_k\} \subset P$ then **T** is said to be a pattern, and in the context of conflict theory **T** is said to be an attainable pattern if there exists an **A** (a real $n \times n$ matrix) such that there are precisely k **ESS**'s for **A**