UNIFORM CONVERGENCE OF MARTINGALES IN THE ONE-DIMENSIONAL BRANCHING RANDOM WALK

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Abstract

In the supercritical branching random walk an initial person has children whose positions are given by a point process $Z^{(1)}$. Each of these then has children in the same way, with the positions of children in each family, relative to their parent's, being given by independent copies of $Z^{(1)}$, and so on. For any value of its argument, λ, the Laplace transform of the point process of n^{th} generation people, normalized by its expected value, is a martingale, the usual branching process martingale being a special case. Here it is shown that under certain conditions these martingales converge uniformly in λ, almost surely and in mean. A consequence of this result is that the limit is, in an appropriate region, analytic in λ.

1. Introduction. This paper considers the one dimensional supercritical branching random walk. The process starts with a single initial ancestor at the origin. She has children, forming the first generation, with their positions on the real line, R, being given by a point process $Z^{(1)}$. Each of these children then has offspring in a similar way, with the positions of each new family relative to their parent being given by independent copies of $Z^{(1)}$. This gives the point process of second generation individuals, denoted by $Z^{(2)}$. Subsequent generations are formed similarly, yielding $Z^{(n)}$ as the n^{th} generation point process. Let $\{ z_r^{(n)} : r\}$ be an enumeration of the positions of the n^{th} generation people.

Let μ by the intensity measure of $Z^{(1)}$ then, as is well known, μ^{n*} (the n-fold convolution of μ) is the intensity measure of $Z^{(n)}$. As the process is supercritical we have $\mu(R) > 1$. Let $m(\lambda)$ be the Laplace transform of μ. Then

$$\begin{aligned} m(\lambda) &= \int e^{-\lambda x} \mu(dx) \\ &= E\int e^{-\lambda x} Z^{(1)}(dx) \\ &= E\sum_r e^{-\lambda z_r^{(1)}} \end{aligned}$$

and hence