ACCUMULATION POINTS OF A PARTICULAR NORMALIZED RANDOM WALK

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Abstract

It is shown that a certain hypothesis is unnecessary when obtaining a generalized one-sided Law of the Iterated Logarithm. Also, as in the independent and identically distributed case the [almost certain] set of accumulation points is an interval. It turns out that our set of limit points is the entire interval $[c, \infty]$, for some finite nonzero constant c.

Results. Herein we establish the almost sure set of accumulation points of normalized weighted sums of independent copies of an unbounded asymmetrical random variable. The random variables under examination are of a particularly peculiar type. They permit the existence of a [weighted] Strong Law of Large Numbers, where the limit is not the common expectation (which may not even exist). We will only consider random variables with either mean zero or without finite first absolute moment. For a more extensive survey of previous results see Adler (1989) and Adler (1990). From Adler and Rosalsky (1989) it is clear that the weights $\{a_n, n \geq 1\}$ should satisfy $n|a_n| \uparrow$ and $\Sigma_{k=1}^n |a_k| = O(n|a_n|)$.

So, as in Adler (1990) we consider weights of the form $a_n = n^\alpha$, $\alpha > -1$.

Let $\{X, X_n, n \geq 1\}$ be i.i.d. random variables and $S_n = \Sigma_{k=1}^n k^\alpha X_k$ denote our partial sums. Define

$$\tilde{\mu}(x) = \int_x^\infty P\{|X| > t\}\, dt \quad \text{if} \quad EX = 0$$

and

$$\mu(x) = \int_0^x P\{|X| > t\}\, dt \quad \text{when} \quad E|X| = \infty.$$

Our hypotheses are $E(X^-/\tilde{\mu}(X^-)) < \infty$ and $\tilde{\mu}(x) \sim \tilde{\mu}(x \log_2 x)$ when the mean is zero. Similarly we assume that $E(X^-/\mu(X^-)) < \infty$ and $\mu(x) \sim \mu(x \log_2 x)$ when $E|X| = \infty$. (It is customary to define $\log \log x$ as $\log_2 x$.)

Next, we establish the sequence $\{c_n, n \geq 1\}$ by way of