WHEN DID JOE'S GREAT . . . GRANDFATHER LIVE? OR: ON THE TIME SCALE OF EVOLUTION.

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Abstract

In a general supercritical branching process the distribution is found of the time back to the birth of the n:th father of a random individual. For single-type processes a probability of mutation (at a site of a specified gene) is introduced. With the infinite alleles interpretation that every mutation occurring is unique the class of individuals with the same allele can be viewed as a generalized "macroindividual". Thereby the question of when the first of a series of n mutations occurred is reduced to finding the birth time of the n:th father in the process of macroindividuals. This is done.

1. Introduction The three great problems of population dynamics are the extinction, the growth, and the composition of populations. The first makes sense already in the simplest of branching processes, the Galton-Watson model, which was indeed born out of it. The second requires a physical time, not only generation counting, and for the third problem one should have access also to the birth order of siblings and times between their births, and preferably a possibility to discern between individuals of different inherited types.

This leads to general, multi-type branching processes (Jagers and Nerman 1984 (the one-type case), Nerman 1984, Jagers 1989 (abstract type spaces)). When an individual is born it inherits a *type*, s, from a *type space*, S with a σ-algebra S. The type, which you may think of as a genotype, determines a probability kernel, $P(s,\cdot)$, over a *life space*, (Ω, A), of all possible life careers. The life space may be very rich in order to contain all relevant aspects of individual life, and is best thought of as an abstract measurable space. What has to be defined on it - otherwise there would be no population to talk of - is a *reproduction process* for individuals. It will be denoted by ξ and viewed as a point process on $S \times \mathbf{R}_+$, the first coordinate of any point yielding the type of that child, the second the father's age at the birth[1]. The processes is supposed finite on bounded subsets of $\mathbf{R}_+$, so that the children can be numbered in birth order (arbitrarily for twins). If σ_k, τ_k are thus the type of the k:th child and father's age at his birth, then