A CENTRAL LIMIT THEOREM FOR EVOLVING RANDOM FIELDS

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Abstract

A functional central limit theorem is proved for certain random fields whose domain has both a temporal and a spacial component. These processes are made up of dependent summands which are measurable with respect to an increasing filtration. Temporal limit theory for semi-martingales is utilized to provide spacial finite-dimensional convergence. Consequently the limiting random fields have independent increments in time, and can be thought of as evolving random fields. In deriving a tightness result the notion of majorizing measures is employed to allow local spacial variability. Thus a functional central limit theorem for evolving random fields with a rather general dependence structure is given here for the first time. Comparison with results available for empirical processes suggest that this result is close to optimal.

Introduction. In this paper we consider conditions for weak convergence of a sequence of random fields which are individually evolving over time. For motivation though, let's first examine a given sequence of random fields at some fixed time point. Let **X** be an arbitrary index set and for each $n \geq 1$, let $\{Y_{n,i}(x) : 1 \leq i \leq n,\ x \in X\}$ be a sequence of random functions $Y_{n,i} : X \to \mathbb{R}$. Let

$$S_n(x) = \sum_{i \leq n} Y_{n,i}(x) \quad \text{for} \quad x \in X.$$

Thus the sequence $\{S_n(x) : n \geq 1,\ x \in X\}$ is a sequence of random fields. We are interested in general conditions under which S_n converges weakly. We wish to allow dependence among the individual summands, the $Y_{n,i}$'s. We do this by considering conditions of the martingale type, involving only conditional first and second moments. (These are more desirable than mixing type conditions which involve the entire distribution.) In addition, we must constrain our indexing family X to satisfy a complexity condition involving the notion of majorizing measures (or, alternatively, metric entropy). There will be a natural topology on **X** associated with our problem.

In two papers Goldie and Greenwood (1986a, b) found conditions for weak convergence of sequences of set-indexed random fields. An unexpectedly difficult aspect of the problem was the characterization of the limiting distribution in