# A LOOK AT PERTURBATION APPROXIMATIONS FOR EPIDEMICS

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**Abstract**

Perturbation-type approximations have been derived by many authors for epidemics and related problems. Here we use the standard perturbation technique familiar to physicists to approximate the mean and variance of paths of some epidemic processes. For epidemics with no removals the saddlepoint approximation to the distribution of infectives is highly accurate for quite small populations and can be used to assess the accuracy of perturbation approximations. For epidemics with removals no saddlepoint approximation is available and simulations have to be used for comparison. The technique turns out to be equivalent to Daley and Kendall's method of 'diffusion of arbitrary constants'.

**1. Introduction.** Approximations of the perturbation type have been around in epidemic theory for a considerable time. Bartlett (1956) used such an approximation to study the fluctuations about the endemic equilibrium in a recurrent epidemic. Bailey (1968) and Weiss (1971) obtained by different routes essentially the same approximations for the mean and variance of a simple epidemic in large populations. Barbour (1972) developed the method of 'diffusion of arbitrary constants' introduced by Daley and Kendall (1965) to extend these results to more general epidemics. McNeil and Schach (1973) considered diffusion approximations of the Ornstein-Uhlenbeck type for epidemic and similar processes. Daniels (1960) described what is essentially a perturbation technique for approximating to the distribution and moments of processes of the epidemic type. Other references may be found in Bailey (1972) and Renshaw (1986).

There is, however, a standard procedure familiar to physicists and engineers for deriving perturbation approximations which can with advantage be used in problems of this kind. My interest in the method was first aroused by Bellman's elegant little monograph (Bellman, 1964) though he treats there only deterministic problems. In the present paper the procedure is applied systematically to obtain perturbation approximations for the mean and variance of some epidemic processes.

**2. The Univariate Case.** We first consider the univariate birth process which includes the so-called simple epidemic with no removals and its generalizations. At time $t$ there are $N(t)$ infectives and the probability of a new infection