ON DIRECTED POLYMERS IN A RANDOM ENVIRONMENT

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We consider random walks whose laws are perturbed in an irregular way by a second random mechanism, the so-called random environment. The perturbation acts in such a way that visits to certain randomly chosen points are favoured or disfavoured. There have been proposed several models in the mathematical physics literature, sharing the common feature that only very few facts are known on a rigorous mathematical level. We present here some of these models and some problems connected with them.

We always look at a random walk on the d-dimensional lattice $\mathbf{Z}^d$ but we start by introducing the random environment, the "disorder". It is given by i.i.d. random variables $X(i)$, $i \in \mathbf{Z}^d$, satisfying

$$X(i) > 0 \text{ almost surely} \tag{1}$$

$$EX(i) = 1. \tag{2}$$

$X = 1$ is then just the case where no perturbation occurs. Sometimes, it is convenient to have a one-dimensional parameter $\beta \in \mathbb{R}$ regulating the amount of disorder. This can be done by considering

$$X_\beta(i) = e^{\beta Y(i)}/m(\beta)$$

where $Y(i)$ are i.i.d. real random variables such that $m(\beta) = E(e^{\beta Y}) < \infty$ for β in a neighborhood of 0.

The unperturbed random walk is an ordinary random walk $\xi_0 = 0, \xi_1, \ldots, \xi_T$ on $\mathbf{Z}^d$ whose jump distribution is given by $p(x)$, $x \in Z^d$, $\Sigma_x p(x) = 1$, i.e. we have

$$P(\xi_1 = i_1, \ldots, \xi_T = i_T) = 1_0(\xi_0) \prod_{j=1}^{T} p(i_j - i_{j-1})$$

where $i_0 = 0$. We always assume that for some $\varepsilon > 0$, we have

$$\sum_{x \in Z^d} e^{\varepsilon|x|} p(x) < \infty \quad \text{and} \quad \Sigma\, xp(x) = 0.$$

This random walk and the random environment are chosen to be independent.