

TWO BASIC PARTIAL ORDERINGS FOR DISTRIBUTIONS DERIVED FROM SCHUR FUNCTIONS AND MAJORIZATION

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Abstract

Researchers in applied fields have long recognized the usefulness of inequalities when exact results are not available. The use of inequalities allows us to say that one estimate is better than another, that one maintenance policy is better than another or that a certain selection procedure is better than another etc., even though, we may not know the best estimator, the best maintenance policy or the best selection procedure. Such results are generally obtained from inequalities between two probability measures or random variables. Inequalities between random variables are in turn obtained from deterministic inequalities or deterministic partial orderings.

Hardy, Littlewood and Pólya (1952) in their classical book entitled *Inequalities* have discussed various partial orderings in R^n , one of which is known as majorization. Majorization is intimately related to Schur functions. This partial ordering was used to derive the partial orderings of stochastic majorization and DT ordering among distributions in a series of papers by Proschan and Sethuraman (1977); Nevius, Proschan and Sethuraman (1977); Hollander, Proschan and Sethuraman (1977); and Hollander, Proschan and Sethuraman (1981). Even though many more partial orderings of this type have been studied in recent papers and books by Marshall and Olkin (1979), Tong (1980), Boland, Tong and Proschan (1987, 1988), Abouammoh, El-Newehi and Proschan (1989), the above two partial orderings remain the centerpiece in this type of research endeavor. In this expository paper, we describe the essentials of stochastic majorization and DT ordering and demonstrate some applications. A new proof of a slight generalization of earlier result on DT functions in Hollander et al., 1981 is given.

Introduction

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