SUFFICIENCY

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Introduction

We undertake with this title a brief survey of various definitions of *sufficiency*, with some of their properties and relationship between them.

Works on this theme are found in a sequence, if not so much as a stream, of developments from the sixties through eighties. We consider such works as attempts at mathematical conceptualization of the statistical notion of sufficiency, and try to examine how far they have been successful in capturing the intuitive and logical content of the notion. Emphasis has been naturally put on the more recent developments, but some earlier results had to be touched upon as long as they make a part of historical or logical background.

A reason for this choice of a theme is that sufficiency today is not as prolific a subject as in early days, making it difficult to draw a *recent trend* out of the publications in last few years. Only a few titles with the word *sufficiency* appear each year in Current Index to Statistics, mostly with their main interest in neighboring though closely related subjects, e.g., ancillarity, information and comparison of experiments. They will be better treated separately under the respective titles, rather than thrown together into such a short survey as this one.

Out of the remaining papers in sufficiency proper, being still fewer in number, we could pick out some fairly recent results to form an additional section on *Basu Theorems*.

Neither a monograph nor a bibliography on this subject recently came into our attention. So the early bibliography by Basu & Speed (1975) as well as the survey *Partial sufficiency* (Basu, 1978) is still partially sufficient (at least) to a reader.

Statistical Notion and Mathematical Definitions

Sufficiency as a statistical notion means the property of a statistic retaining all the relevant information contained in the whole sample. As is well known, it first appeared in Fisher (1920) (see Stigler, 1973, for historical background) which pointed out that an estimate of a parameter can be regarded to sum up the whole of the information respecting the parameter which a sample provides if, for any of its given value, the conditional distribution of any other estimate is independent of the parameter. This idea of expressing the notion by means of conditional probability developed into Fisher's (1922) first definition of sufficiency. A statistic T is called sufficient if:

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