UNBIASED SEQUENTIAL BINOMIAL ESTIMATION

Bimal K. Sinha, University of Maryland-Baltimore County,

and

Bikas K. Sinha, Indian Statistical Institute

Abstract

We review the literature on unbiased estimation of some functions of the Bernoulli parameter p in the sequential case. Connections between the so-called efficient and inefficient sampling plans through the well known concept of sufficiency which have been explored recently are also presented.

Introduction

Under the set up of independent identical Bernoulli trials with parameter p, various aspects of unbiased estimation of a parametric function g(p) have been studied in the literature. Early works of Girshick, Mosteller and Savage (1946), Wolfowitz (1946, 1947), Lehmann and Stein (1950), De Groot (1959) and Wasan (1964) are devoted to some general results on sequential binomial estimation. Later works by Gupta (1967), Sinha and Sinha (1975), Sinha and Bhattacharya (1982) and Sinha and Bose (1985) deal with problems related to unbiased estimation of $1/p$. Recently Bose and Sinha (1984) studied the connections between the so-called efficient and inefficient Bernoulli sampling plans through the well known concept of sufficiency of statistical experiments.

Our object in this paper is to present a comprehensive review of most of the available results in this area. We omit proofs of all the results. However, detailed and exact references to various results are provided.

The next section is devoted to setting up the notations, nomenclature, and definition of efficient sampling plans. In the third section, we provide results on efficient sampling plans. The problem of unbiased estimation of $1/p$, which has received considerable amount of attention in the literature, is discussed in fourth section. In fifth section, we discuss the connection between efficient and inefficient sampling plans via the concept of sufficiency. Some concluding remarks are made in the last section.

Notations and Nomenclature

Let $(Z_i,\ i = 1, 2\ldots)$ be an i.i.d. sequence of Bernoulli variates with $P(Z_i = 1) = p$ and $P(Z_i = 0) = 1 - p = q$ (say). We assume $p \in \Omega \subseteq (0, 1)$. Any realization of this process can be exhibited as a lattice path in the (X, Y)-plane, where a particle moves from the origin one step to the right (along X-axis) if the incoming observation is 0 and one step above (along Y-axis) if it is 1. A