THE PITMAN CLOSENESS OF STATISTICAL ESTIMATORS: LATENT YEARS AND THE RENAISSANCE

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Abstract

The Pitman closeness criterion is an intrinsic measure of the comparative behavior of two estimators (of a common parameter) based solely on their joint distribution. It generally entails less stringent regularity conditions than in other measures. Although there are some undesirable features of this measure, the past few years have witnessed some significant developments on Pitman-closeness in its tributaries, and a critical account of the same is provided here. Some emphasis is placed on nonparametric and robust estimators covering fixed-sample size as well as sequential sampling schemes.

Introduction

In those days prior to the formulation of statistical decision theory (Wald, 1949), the reciprocal of variance [or mean square error (MSE)] of an estimator (T) used to be generally accepted as an universal measure of its precision (or efficiency). The celebrated Cramér-Rao inequality (Rao, 1945) was not known that precisely although Fisher (1938) had a fair idea about such a lower bound to the variance of an estimator. The use of mean absolute deviation (MAD) criterion as an alternative to the MSE was not that popular (mainly because its exact evaluation often proved to be cumbersome), while other loss functions (convex or not) were yet to be formulated in a proper perspective. In this setup, Pitman (1937) proposed a novel measure of closeness (or nearness) of statistical estimators, quite different in character from the MSE, MAD and other criteria. Let T_1 and T_2 be two rival estimators of a parameter θ belonging to a parameter space $\Theta \subseteq R$. Then T_1 is said to be closer to θ than T_2, in the Pitman sense, if

$$P_\theta\{|T_1 - \theta| \leq |T_2 - \theta|\} \geq 1/2, \forall \theta \in \Theta, \tag{1}$$

with strict inequality holding for some θ. Thus, the Pitman-closeness criterion (PCC) is an intrinsic measure of the comparative behavior of two estimators. Note that in terms of the MSE, T_1 is better than T_2, if

$$E_\theta(T_1 - \theta)^2 \leq E_\theta(T_2 - \theta)^2, \forall \theta \in \Theta, \tag{2}$$

with strict inequality holding for some θ; for the MAD criterion, we need to replace $E_\theta(T - \theta)^2$ by $E_\theta|T - \theta|$. In general, for a suitable nonnegative loss function $L(a, \theta) : R \times R \to R^+$, T_1 dominates T_2 if