ANCILLARITY

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Introduction

A statistic is ancillary if its distribution does not depend on the parameters of the model. It might appear at first sight as if ancillary statistics could make no contribution to inference about these parameters. However, as was pointed out by Fisher who first defined and named the concept (1925, 1934, 1935, 1936), this appearance is deceptive. By themselves ancillaries of course carry no information about the parameters, but they may be very useful in conjunction with other parts of the data.

Ancillarity has connections with many other statistical concepts, among them sufficiency, group families, conditionality, completeness, information, pre-randomization, and mixtures. Its most important impact on statistical methodology comes from the suggestion that inference should be carried out conditionally given an ancillary statistic rather than unconditionally. For small samples, the resulting conditional procedures can be less efficient than their unconditional counterparts; however, they have the advantage of greater relevance to the situation at hand and frequently are simpler. Typically, the efficiency difference tends to disappear as the sample size becomes large (see for example Barndorff-Nielsen, 1983, and Liang, 1984).

Since ancillaries typically are not unique, the recommendation to condition on an ancillary is not sufficiently specific. Conditioning comes closest to its purpose of making the inference relevant to the situation at hand if the ancillary is maximal, i.e. if there exists no other (nonequivalent) ancillary of which it is a function. The concept of maximal ancillary, which is basic to the theories of ancillarity and conditioning, was introduced by Basu (1959) who showed that maximal ancillaries always exist,[2] but noted that even they may not be unique. In the same paper he also pointed out some measure theoretic complications which require the slightly weaker definition of essential maximality for their resolution. Further results and some basic examples were given in Basu (1964) and some additional generalizations in Basu (1967).

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[2]For a more precise statement see Theorem 3.