DEPENDENCE ORDERING IN STATISTICAL MODELS AND OTHER NOTIONS

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The relation of notions of dependence ordering with other notions in statistics such as heaviness of tail and largeness of dispersion are reviewed and developed. The problem of overdispersion in reproductive studies is discussed as a practical, attractive example.

1. Introduction. The notion of positive dependence is closely associated with other various statistical notions. This has been emphasized by many authors including Yanagimoto and Sibuya (1972) and Karlin and Rinott (1980a). Recent theoretical development of the notion of positive dependence permits us better understandings of statistical models. However, as Kimeldorf and Sampson (1987) stressed, the research on dependence orderings does not appear fully developed. We often assume a family of distributions having monotone dependence. To state this clearer, consider a distribution function, $F_\alpha(x)$, of a random variable X on R^n. The suffix α, representing the degree of largeness of dependence, is considered to be favorably parameterized, if $\alpha = 0$ stands for independence and dependence ordering is monotone increasing in α; as a result $\alpha > 0$ means positive dependence of $F_\alpha(x)$.

The situation is largely different according to the value of n. In the bivariate case negative dependence of (X_1, X_2) is reasonably recognized as positive dependence of $(X_1, -X_2)$. The notion of negative dependence is much more complicated in the multivariate case than in the bivariate case. The aim of the present paper is to review and develop dependence orderings with emphasis on the relation with other statistical notions and practical models. In Sections 2 and 4 definitions of dependence orderings are studied in the bivariate case and in the multivariate case. Some relations are discussed in Sections 3 and 5.

As usual conventions, we will employ simple descriptions unless any confusion is anticipated. Therefore, for example, dependence of a distribution, that of a random variable with the distribution and that of a distribution function of the distribution are not distinguished, and an increasing function means a nondecreasing function.

2. Dependence Ordering: Bivariate Case. A systematic definition of notions of positive dependence in the bivariate case was developed by Lehmann

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