A STRONG LIMIT THEOREM FOR PROCESSES WITH ASSOCIATED INCREMENTS

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A finite collection of random variables $X_1, \ldots, X_m$ is said to be associated if for any two coordinatewise nondecreasing functions f, g on R^m

$$\mathrm{Cov}(f(X_1, \ldots, X_m), g(X_1, \ldots, X_m)) \geq 0$$

whenever the covariance is defined; a stochastic process $X(t)$ is said to have associated increments if for each t, $X(t)$ and the increments of the process in (t, ∞) are associated.

THEOREM. *If $X(t)$ is a separable, mean zero stochastic process with associated increments, and $H(t) \uparrow \infty$ is positive and continuous and such that $\int_0^\infty \frac{d\sigma(t)}{H(t)} < \infty$, where $\sigma(t) =$ standard deviation $(X(t))$, then $\frac{X(t)}{H(t)} \to 0$ a.s.*

1. Introduction. Let $\{X_j, 1 \leq j \leq m\}$ be a collection of random variables. The collection is said to be associated if for any coordinatewise non-decreasing functions f, g on R^m, $\mathrm{Cov}(f(X_1, \ldots, X_m), g(X_1, \ldots, X_m)) \geq 0$ whenever the covariance is defined; an infinite collection is associated if every finite subcollection is associated. In Newman and Wright (1982) it was shown that associated random variables satisfy several of the classical martingale inequalities.

In this paper we consider continuous time stochastic processes. A stochastic process $X(t)$ is said to have associated increments if for each t, $X(t)$ and the increments of the process in (t, ∞) are associated. Such processes have many of the sample function properties that a separable submartingale has. Many of these properties are discussed in Wood (1983), in the more general case of (continuous time) demimartingales. These properties also hold for separable and centered processes with independent increments. In our case we assume the process is separable, mean zero, with *associated* increments. One interesting consequence shown here is that a separable, mean zero process with associated increments is automatically centered.

The properties above allow us to prove the following strong limit theorem: Let $\{X(t), t \geq 0\}$ be a separable, mean zero process with associated increments. If

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