MODELS FOR DEPENDENT LIFELENGTHS INDUCED BY COMMON ENVIRONMENTS

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Multivariate distributions for the lifelengths of the components of a system operating under a common environment, when the environment has a different effect on each component, and when the environment is dynamic, are derived. Modelling of the dynamic environment is by a gamma process.

1. Introduction. Multivariate distributions for the lifelengths of biological and engineering systems have been proposed by Freund (1961), Downton (1970), Marshall and Olkin (1967), Lindley and Singpurwalla (1986), and Lee and Gross (1990). In this paper we build upon the theme proposed by Lindley and Singpurwalla, and generate classes of multivariate distributions which may lead to improved assessments of system reliability.

As a motivating scenario, suppose that we have an m-component, parallel redundant system, and suppose that the lifelengths of these components are judged exponential with known scale parameters $\lambda_{10}, \lambda_{20}, \ldots, \lambda_{m0}$ when they are tested in a laboratory individually. The λ_{io}'s [or more generally, the $\lambda_{io}(t)$'s, if the lifelengths are judged to be other than exponential] will be referred to as the *baseline failure rates* of the m components. Suppose that the effect of the common operating environment—when assumed to be static over time—is to modulate each λ_{io} by a common factor η, where η is unknown and has distribution G, so that the reliabilities become $\exp\{-\int_0^t \eta\lambda_{io}(u)du\}$. Uncertainty about η induces dependence among the component lifelengths $T_1, \ldots, T_m$. The T_i's, $i = 1, \ldots, m$, have a multivariate distribution whose nature is prescribed by the form of G. When the operating environment is dynamic, η becomes a function of time t, say $\eta(t)$; we will refer to η, or more generally $\eta(t)$, $t \geq 0$, as the *environmental factor function*—henceforth EFF. It is important to bear in mind that the EFF is merely a parameter that has little, if any, physical meaning. It is introduced for convenience with the aim of capturing our opinion about the effects of the environment on the failure rate of each component.

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