DYNAMIC CONSTRUCTION AND SIMULATION OF RANDOM VECTORS

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In this paper described is a novel method of generation of nonnegative random variables $T_1, \ldots, T_n$ which may be dependent and which have an absolutely continuous joint distribution. In this method first $\min(T_1, \ldots, T_n)$ is generated and then one of the indices $1, \ldots, n$ (j_1, say) is chosen and T_{j_1}, is determined. Once $T_{j_1}, \ldots, T_{j_k}$ have been determined, then $\min_{j \in \{1,\ldots,n\} - \{j_1,\ldots,j_k\}}(T_j)$ is generated and one of the remaining indices (j_{k+1}, say) is chosen and $T_{j_{k+1}}$ is determined. The novel method has a clear intuitive meaning, mainly for applications in reliability theory. The new method is applied to obtain stochastic comparisons of two absolutely continuous random vectors consisting of nonnegative random variables. Also, the use of the new method is illustrated in obtaining some multivariate aging properties and positive dependence properties of vectors of random lifetimes.

1. Introduction. Consider an absolutely continuous nonnegative random variable T with distribution function F, survival function $\bar{F} = 1 - F$ and hazard function $\Lambda = -\log \bar{F}$. The random variable T can be thought of as a lifetime of a device. The hazard rate (or the instantaneous failure rate of the device) at time t is defined as

$$\lambda(t) = \frac{f(t)}{P(T \geq t)} = \frac{f(t)}{\bar{F}(t)} = \frac{d}{dt}\Lambda(t), \ t \geq 0,$$

where $f = \frac{d}{dt}F$ is the density function of T. It is well known (and easy to verify) that T is stochastically equal to (that is, has the same distribution as) the time of the first epoch of a nonhomogeneous Poisson process on $[0, \infty)$ with intensity function λ. Thus, in order to generate a random variable $\hat{T}$ which has the same

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