

INTERRELATIONS AMONG VARIOUS DEFINITIONS OF BIVARIATE POSITIVE DEPENDENCE

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In this paper—based on an extensive computer simulation—a detailed investigation and comparisons of seven types of positive dependence properties appearing in statistical and reliability literature is presented.

A numerical index of the “strength” of positive quadrant dependence (PQD) is proposed and compared with the correlation coefficient. This index can also be adopted to various other definitions of dependence.

1. Introduction. Dependence relations among variables constitute one of the basic topics of applied probability and statistics. This theory goes back to the classical investigations of Pearson in (1900) and (1904). While the concept of independence is mathematically defined by an equality relation, the violation of this equality by definition signifies dependence. Difficulties to provide an adequate measure of dependence are illustrated by the following statement of Cramér (1924): “Every attempt to measure a conception like this by a single number must necessarily contain amount of arbitrariness and suffer from certain inconveniences.” In fact, Pearson (1900, 1904), Gini (1914), Fréchet (1951), Cramér (1924), Hoeffding (1940), Rényi (1959), Kolmogorov (1933), Lehmann (1966), and Lai and Robbins (1976) should be mentioned among the leading statisticians and probabilists who have studied this problem. The more recent work of Lehmann (1966) triggered an additional spurt of activity in this area after a certain period of dormancy and, in the last decade, we are witnessing a burgeoning awakening in this field which is closely associated with the renewed interest in statistical and probabilistic reliability methodology pioneered by the works of Barlow and Proschan (and summarized in their monograph (1981)).

A survey of results up to 1975 is presented in the paper by Kotz and Soong (1977) where some 10 properties of positive dependence have been discussed and

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