INFORMATION, CENSORING, AND DEPENDENCE

BY MYLES HOLLANDER[1], FRANK PROSCHAN[1], AND JAMES SCONING[1]

Florida State University, Florida State University, and University of Iowa

Hollander, Proschan, and Sconing (1987) used the theory of majorization to develop and study various information measures in the randomly right-censored model where the basic observation is $\underline{Z} = (Z, \delta)$ where $Z = \min(X, Y)$, X is the survival time, Y is the censoring time, Y is assumed to be independent of X, and $\delta = 1$ if $X \leq Y$, $= 0$ otherwise. Here we use coefficients of divergence to derive measures of how dissimilar the joint distribution of $(X, \underline{Z})$ is from the product of its marginals. These measures contain some of the HPS information measures as special cases. We also introduce various concepts of bivariate dependence to measure the degree to which Y inhibits the ability to see X.

1. Introduction and Summary. Consider the randomly censored model where X is the survival time, Y is the censoring time, and where Y is assumed to be independent of X. We observe (Z, δ) where $Z = \min(X, Y)$, $\delta = I(X \leq Y)$, where $I(A)$ denotes the indicator of the event A. Hollander, Proschan, and Sconing (1987) [hereafter referred to as HPS (1987)] used the theory of majorization to develop and study various measures for this model.

One of the measures developed by HPS (1987) for the case where X and Y are discrete is a generalization of Shannon's (1948) information in the uncensored case.

DEFINITION 1.1. For the censored model where X and Y have discrete distributions $p_i = \Pr(X = i), q_i = \Pr(Y = i)$, the information in the experiment (X, Y) is defined to be

$$(1) \qquad H(X, Y) = H(\underline{p}, \underline{q}) = -\sum_i q_i \left[\sum_{j \leq i} p_j \log p_j + \bar{P}_{i+1} \log \bar{P}_{i+1} \right]$$

[1]Research sponsored by the Air Force Office of Scientific Research, AFSC, USAF, under Grant AFOSR 88-0040. The U.S. Government is authorized to reproduce and distribute reprints for governmental purposes notwithstanding any copyright notation thereon.

AMS 1980 subject classifications. Primary 62N05; secondary 62B10.

Key words and phrases. Bivariate dependence, coefficients of divergence, association.

We are grateful to Professor Ian McKeague for helpful conversations about information. We also thank the referee and editors for helpful comments.