RELATIVE ERRORS IN RELIABILITY MEASURES

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A common assumption, in reliability and lifetesting situations when the components are installed in series system, is that they are independent and are exponentially distributed. In this paper we study the relative error in reliability measures such as the reliability function, the failure rate and the mean residual life under the erroneous assumption of independence when in fact lifetimes follow a bivariate exponential model. The behavior of these errors is discussed to examine their structure as a function of time. Some of the existing results in the literature follow as special cases.

1. Introduction. Klein and Moeschberger (1986, 1987) have studied the relative error (defined in Section 3) in system reliability and system mean life when the components follow the bivariate exponential distributions of Marshall and Olkin (1967), Freund (1961), Gumbel (1960), Downton (1970), and Oakes (1982). Moeschberger and Klein (1984) have studied the relative error in the Gumbel II bivariate exponential model.

In this paper we consider a series system whose components follow bivariate exponential models. The joint distribution of the component lives may not be uniquely determined from the observable data on (T, I), where $T = \min(X_1, X_2)$ and $I = I_{\{X_1 < X_2\}}$, a problem of nonidentifiability as described by Tsiatis (1975) and others. If the data on T shows that T has an exponential distribution, then the component lives can be assumed to follow any one of the models described by Marshall and Olkin (1967), Freund (1961), and Block and Basu (1974). If the data shows simultaneous failure of both the components, the shock model developed by Marshall and Olkin will be more appropriate. If T is not exponential but the marginals are exponentials with the same parameters as for the independent case, then one may assume either one of Gumbel I or Gumbel II (1960). If in fact, the joint lifetimes follow any one of the five models mentioned above, the assumption of independence will lead to inappropriate conclusions.

[1]Partially supported by NSERC Research Grant A–4850.

AMS 1980 subject classification. Primary 62N, 62P.

Key words and phrases. Failure rate, reliability function, mean residual life, bivariate exponential distributions.