UNBIASEDNESS OF TESTS OF HOMOGENEITY WHEN ALTERNATIVES ARE ORDERED

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Let $X_1, X_2, \ldots, X_k$ be independent random variables whose densities are from an exponential family with parameters $\theta_1, \theta_2, \ldots, \theta_k$, respectively. That is, the densities are $f(x_i \mid \theta_i) = c(\theta_i)e^{x_i\theta_i}g(x_i)$. Assume that g is a Polya frequency function of order two (PF_2). Consider testing the null hypothesis $H : \theta_1 = \theta_2 = \ldots = \theta_k$ vs. the alternative $K : \theta_1 \geq \theta_2 \geq \ldots \geq \theta_k$. Write $\mathbf{x} = (x_1, x_2, \ldots, x_k)$ and define a partial ordering $>>^*$ on $\Re^k$ by $\mathbf{x} >>^* \mathbf{y}$ if and only if $\Sigma_{i=1}^j x_i \geq \Sigma_{i=1}^j y_i$ for $j = 1, 2, \ldots, k-1$ and equality for $j = k$. A function $\varphi(\mathbf{x})$ is said to be ISO* if $\mathbf{x} >>^* \mathbf{y}$ implies $\varphi(\mathbf{x}) \geq \varphi(\mathbf{y})$. We prove that if $\varphi(\mathbf{x})$ is a similar test which is ISO* then φ is unbiased. In fact if $\varphi(\mathbf{x})$ is ISO* the power function of the test is conditionally monotone nondecreasing along rays orthogonal to the equiangular line. For cases where the distribution satisfies the semi-group property the power function is unconditionally monotone along these rays. Furthermore a way to generate unbiased tests with monotone power is given.

The result contrasts with and complements the result of Robertson and Wright (1982). They prove that when the density has the semi-group property (normal and Poisson, for example) the tests which are ISO* have ISO* power functions. Such a finding is different from ours. The class of distributions for which our result holds is larger than the class in Robertson and Wright.

Applications for particular distributions and particular tests are given. Also some admissibility results are given for particular distributions. For example, it is proven that Bartholomew's test is admissible for the normal case.

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