TIME SERIES MODELS FOR NON-GAUSSIAN PROCESSES

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In this paper we present univariate and multivariate time series models for processes with non-Gaussian marginal distributions. These include bivariate autoregressive-type models for processes with bivariate exponential marginals, nonlinear autoregressive-type models for processes with Dirichlet marginals, and nonlinear models for univariate time series with arbitrary marginal distributions. Examples of applications to real data sets are given for some of the models discussed. When applicable, the theory of positive dependence is used to establish the association of the processes.

1. Introduction. The classical model in multivariate time series analysis is the $m \times 1$ vector linear process given by

$$\mathbf{X}(n) = \Sigma_{j=-\infty}^{\infty} A(j)\epsilon(n-j), \quad n \in \mathcal{Z} \tag{1}$$

where $\mathcal{Z} = \{0, \pm 1, \pm 2, \ldots\}$, $\{\epsilon(n), n \in \mathcal{Z}\}$ is a sequence of iid $m \times 1$ random vectors with mean zero and unknown covariance matrix, and $\{A(n), n \in \mathcal{Z}\}$ is a sequence of unknown $m \times m$ matrices such that $\Sigma_{j=-\infty}^{\infty} \| A(j) \| < \infty$ where $\| \cdot \|$ denotes the usual eigenvalue norm. Note that autoregressive (AR), moving average (MA), and mixed autoregressive-moving average (ARMA) models are important particular cases of the classical linear process (1).

If the $\epsilon(n)$'s are Gaussian then clearly so are the $\mathbf{X}(n)$'s in (1). Furthermore, if the $\mathbf{X}(n)$'s are Gaussian with mean zero and absolutely continuous spectrum, then there is a sequence of iid normal mean-zero random vectors $\epsilon(n)$, $n \in \mathcal{Z}$, and a sequence of matrices $A(n), n \in \mathcal{Z}$, such that the two processes $\mathbf{X}(n)$ and

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