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SOME APPLICATIONS OF CANONICAL MOMENTS IN FOURIER REGRESSION MODELS

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This paper applies recent results on canonical moments for the determination of optimal designs for multivariate Fourier regression models. Optimal designs for discriminating between different Fourier regression models can be found explicitly. It is also demonstrated that these designs may be useful in orthogonal series estimation and for testing additivity in nonparametric regression. In contrast to many other optimality criteria for the trigonometric regression model, the discrimination designs are *not* necessarily uniformly distributed on equidistant points.

1. Introduction. Consider a Fourier regression model in q variables

$$g_{2\lambda}(x) = \sum_{\substack{0\le i_1,\dots,i_q\le\lambda \\ 0\le\sum i_j\le\lambda}} \beta_{i_1,\dots,i_q} \prod_{j=1}^{q} C_{i_j}(x_j) + \sum_{\substack{0\le i_1,\dots,i_q\le\lambda \\ 1\le\sum i_j\le\lambda}} \alpha_{i_1,\dots,i_q} \prod_{j=1}^{q} S_{i_j}(x_j) \tag{1.1}$$

where $q, \lambda \in \mathbb{N}, x \in [-\pi,\pi]^q$, $C_k(z) = \cos(kz) \quad (k = 0,\dots,\lambda)$; $S_k(z) = \sin(kz) \quad (k = 1,\dots,\lambda)$ and $S_0(z) = 1$. A simple example is the case $\lambda = 2, q = 2$, where there are 11 regression functions of the form

$$1,\ \cos x,\ \cos y,\ \cos x\cos y,\ \cos(2x),\ \cos(2y),\ \sin x,\ \sin y,\ \sin x\sin y,\ \sin(2x),\ \sin(2y).$$

Functions of this type are frequently used in orthogonal series estimation of a regression function $g : [-\pi,\pi]^q \to \mathbb{R}$ [see e.g. Eubank (1988, p. 66), Müller (1988, p. 21 and the discussion in Section 2)] where the unknown parameters are estimated by least squares. It is well known that the crucial point in the application of these methods is the appropriate choice of the "smoothing parameter" λ or the degree of the regression [see e.g. Hart (1985)].

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