New Developments and Applications in Experimental Design
IMS Lecture Notes – Monograph Series (1998) Volume 34

EXACT DISTRIBUTIONS OF SEQUENTIAL THRESHOLD ESTIMATORS

BY NANCY FLOURNOY AND DOUGLAS G. KELLY

American University and University of North Carolina

We observe that, for any sequential procedure that is designed to estimate a parameter such as the threshold in a binary response experiment, the distribution of any estimator after n steps is discrete, with at most 2^n possible values. Furthermore, this distribution can be computed exactly, rendering simulations unnecessary. We compute and analyze the distributions of some estimators based on a certain up-down procedure, with a view to their dependence on the initial level and stepsize.

1. Introduction. Binary response experiments and psychometric functions. The ideas and methods considered here apply to any binary response experiment; that is, to any experiment having two possible outcomes, denoted 0 and 1, in which the value of a control variable x (set by the experimenter) determines the probability $\Psi(x)$ of response 1. The general design problem is to choose values of x so that the responses will allow efficient estimation of various features of the function $\Psi(x)$.

Such situations arise in a great many scientific areas. For concreteness we consider the context of psychophysics. In this setting, a stimulus can be delivered to a subject at different levels x, and responses 1 and 0 correspond to correct and incorrect identification, respectively, of some feature of the stimulus. For example, a brush may be stroked lightly on the subject's skin, at a fixed, constant velocity and pressure, and the length of skin traversed, as well as the direction of the brush stroke, can be varied. The subject is to attempt to identify correctly the direction of motion, and if the subject's probability of doing so is $\Psi(x)$ when the traverse length (level) is x, then $\Psi(x)$ is the subject's psychometric function. Another example is in hearing tests, where tones of a fixed frequency but different amplitudes x are delivered to a subject's ear, and response 1 corresponds to the tone's being audible to the subject.

In such studies it is assumed that $\Psi(x)$ is a monotone increasing function of x. It is also of course between 0 and 1, and it is usually assumed to be continuous; but it

Received October 1997; revised March 1998.
AMS 1991 subject classifications. Primary 62L12; secondary 62E15.
Key words and phrases. binary responses, sequential methods, up-and-down designs, psychometric functions.