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# Separate Optimum Estimating Function for the Ruled Exponential Family

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## ABSTRACT

A general family of distributions with mutually orthogonal parameters is introduced. The mean parameter is estimated by using the score function, and the dispersion parameter by using the projected estimating function of the score function. Both the estimating functions attain the minimum of the sensitivity criterion due to Godambe (1960, 1976).

**Key Words:** Negative binomial distribution, orthogonality, separate estimating function, unbiasedness

## 1 Introduction

For many distributions in common practice, the parameter has two types of components. One of them represents the mean, and the other the dispersion. The mean is usually estimated by using the score function, which is essentially free from the remaining component. This fact is a theoretical background of the familiar generalized linear model (GLM).

The aims of the present paper are to define a family of distributions having the above properties in a general way, and also to discuss the separate estimation of each component of the parameter.

## 2 Ruled exponential family

Consider first a density function of a random variable $x$ on $R^n$. For simplicity we will not distinguish a random variable and a sample of size 1, unless any confusion is anticipated. Let $t(= t(x))$ be a statistic on $R^s$ with $s < n$, and