Institute of Mathematical Statistics

LECTURE NOTES — MONOGRAPH SERIES

Inference From Stable Distributions

Hammou El Barmi and Paul I. Nelson
Kansas State University

ABSTRACT

We consider linear regression models of the form $\mathbf{Y} = X\boldsymbol{\beta} + \epsilon$ where the components of the error term have symmetric stable ($S\alpha S$) distributions centered at zero with index of stability α in the interval (0,2). The tails of these distributions get progressively heavier as α decreases and their densities have known closed form expressions in only two special cases: $\alpha = 2$ corresponds to the normal distribution and $\alpha = 1$ to the Cauchy distribution. The $S\alpha S$ family of distributions has moments of order less than α. Therefore, for $\alpha \leq 1$, the components of $X\boldsymbol{\beta}$ are viewed as location parameters. The usual theory of optimal estimating functions does not apply since variances of the components of $\mathbf{Y}$ are not finite. We study the behavior of estimators of $\boldsymbol{\beta}$ based on 3 types of estimating equations: (1) least squares, (2) maximum likelihood and (3) optimal norm. The score function from these stable models can also be used to consistently estimate $\boldsymbol{\beta}$ for a general class of variance mixture error models.

Key Words: Stable distribution, regression, estimating function, consistency, constrained minimization, variance mixture.

1 Introduction

Statistical analyses of regression type models of the form

$$\mathbf{Y} = X\boldsymbol{\beta} + \epsilon \tag{1.1}$$

typically assume that the error terms have independent normal distributions with common variance and that the components of the full rank design matrix X are constants. Here, we generalize and allow the components of ϵ to have independent symmetric stable distributions with infinite variance. A stable distribution symmetric about μ has a log-characteristic function of the form

$$\psi(t) = -|\sigma t|^{\alpha} + i\mu t, \tag{1.2}$$